Professor Kenneth J. Arrow and the Theory of Production

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It is a great honor to have this opportunity to speak about Prof. Kenneth Arrow at this Academic Tribute. Ken made path-breaking contributions in almost all fields of economics. As I looked at today's program, with all the special panels, I did wonder what more I could say. Then I decided to talk about the major contributions that Ken made to the theory of production.

First, the Arrow, Chenery, Minhas and Solow (1961) article, "Capital-Labor Substitution and Economic Efficiency," was a "watershed" article and its publication sent the profession off to new directions. It is a masterly synthesis of both theoretical and empirical research. It is a deep and rich article--I always learn something new each time I re-read it. The article demonstrates convincingly that the production technology, whether at the industry or the economy level, is neither the Leontief fixed-coefficient type nor the Cobb-Douglas type, the two most popular assumptions at the time. The elasticity of substitution between capital and labor varies across economies and industries and can only be determined through empirical estimation. Moreover, if the elasticity of substitution is assumed to be a constant with respect to the relative price of capital and labor, it implies that the production function has the C.E.S. form, which Ken and Prof. Robert Solow independently derived. It is of interest that the cost function corresponding to the C.E.S. production function also has the same algebraic form. Of course, if the elasticity of substitution is not a constant equal to unity, that is, the production function is not of the Cobb-Douglas type, the relative factor share is not constant, but depends on the relative factor price in addition to the value of the elasticity of substitution.

The elasticity of substitution is a measure of the curvature of the isoquant. While it may be a constant on each isoquant, it is not necessarily the same constant across isoquants. It can, in principle, change with the quantity of output as the isoquants move out in the northeasterly direction. Constancy of the elasticity of substitution is also independent of the assumption of homotheticity, and in particular, does not require constant returns to scale. This can be most easily seen in a C.E.S. cost function, in which the distribution parameter and even the elasticity of substitution itself can be a function of the quantity of output as well as time. Technical progress can also shift the isoquants and change the elasticity of substitution, and not necessarily in a Hicks-neutral way.

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The value of the elasticity of substitution between capital and labor remains relevant today. Advances in information technology, especially in the areas of artificial intelligence (AI) and robotics, have increased the potential substitutability between capital, broadly defined, and labor. Bear in mind that software (for example, artificial intelligence) is a form of intangible capital and a robot is a form of tangible capital. They can make the isoquants in the capital-labor space flatter and the elasticity of substitution possibly greater than unity in some industries. However, whether an economy will operate in this region still depends on the relative price of capital (including the exceptionally low rate of interest and the falling user cost of AI and robots) and labor. This will also have implications at the macroeconomic level: on output, employment, investment and relative factor share. Over the past decade or so, the share of labor has been declining in the U.S., and in some other developed economies as well. Perhaps the rise of the value of the elasticity of substitution to above unity and the declining user cost of capital may provide a partial explanation.

How would one try to identify empirically a change in the elasticity of substitution over time, at either the industry or the economy level? It will require distinguishing capital by vintage—whatever changes in the elasticity of substitution caused by technical progress can only be embodied and reflected in new capital investment. The accelerated rates of technological obsolescence in recent years may have also hastened the rise in the value of the elasticity of substitution on average. In addition, identifying a change in the value of an elasticity of substitution may also require taking into account the difference between ex-ante and ex-post substitutability, as done by the late Leif Johansen and also suggested by Ken. Ex-ante substitutability is always greater than ex-post substitutability. But the data requirements for empirical identification of ex-ante substitutability are even more formidable, because it requires knowledge of potentially adoptable technologies in addition to the adopted technology at each point in time. This is not likely to be possible if the technology has been undergoing significant changes.

The above discussion leads naturally into the Arrow (1962) article, "The Economic Implications of Learning by Doing." In this article, Ken modeled the effects of learning through the cumulative production of new capital goods—so that the vintage of the capital investment matters. Learning by doing may also be regarded as a form of economies of scale, but over the time dimension rather than the space dimension—cumulative output over time rather than aggregate output over a specified space in a given period. Interestingly, Ken's empirical example of learning by doing, taken from the airframe industry, shows that the amount of labor required to produce the Nth airframe of a given type, N a positive integer, is proportional to N raised to the power minus one-third. This implies that the marginal labor cost of the second unit produced is only 80% of that of the first, and it falls rapidly to half that of the first by the 8^{th} unit produced. There is a similar empirical six-tenths power rule, widely used by engineers, which specifies that capital cost rises in proportion to the output capacity raised to the power two-thirds. These are all very real and significant, but non-homothetic, or biased economies of scale.

The existence of significant learning-by-doing and economies of scale effects, whether at the plant, firm, industry or economy level, is an inconvenient truth. Since both will inevitably lead to an eventual monopoly or quasi-monopoly situation, it sets up a possible conflict between public welfare and private profit. We can see it today in the information and communication technology industries, where network externalities are an important source of economies of scale. It also raises the question of what the optimal government regulatory policy should be. The challenge for us is how to avoid "winner-take-all" but instead try to make all winners.