

# Possible Extensions of Input-Output Analysis

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Lawrence J. Lau 刘遵义

**Ralph and Claire Landau Professor of Economics, The Chinese University of Hong Kong  
and**

**Kwoh-Ting Li Professor in Economic Development, Emeritus, Stanford University**

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Tel: +852 3943 1611; Fax: +852 2603 5230

Email: [lawrence@lawrencejlau.hk](mailto:lawrence@lawrencejlau.hk); WebPages: [www.igef.cuhk.edu.hk/ljl](http://www.igef.cuhk.edu.hk/ljl)

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# Introduction

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- ◆ I am not a specialist in input-output analysis. Everything I know about input-output analysis I learnt from Prof. CHEN Xikang over the years.
- ◆ I want to take this opportunity to express my deep gratitude to Prof. CHEN Xikang for his advice, counsel and guidance, for being an exemplary model of a scholar, and above all, for his friendship, over the past almost four decades.

# Introduction

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- ◆ Input-output analysis is a valuable and useful tool for economic planning and for understanding the sectoral interdependence within an economy. I would like to take this opportunity to suggest some potentially useful extensions of input-output analysis that can enhance its applicability.
- ◆ The typical assumptions of input-output analysis include:
  - ◆ (1) fixed input-output coefficients;
  - ◆ (2) constant returns to scale;
  - ◆ (3) all goods are desirable;
  - ◆ (4) fixed infrastructure; and
  - ◆ (5) homogeneity of goods (in the same sector).
- ◆ It is possible to take into account the deviations from these standard assumptions of input-output analysis.

# The Standard Leontief Input-Output Model of a Closed Economy

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- ◆ The input-output model:  
 $x = Ax + y$ ;  
 $[I - A]x = y$ ;  
 $x = [I - A]^{-1} y$
- ◆ Nonnegativity constraints:  
 $x \geq 0; y \geq 0$
- ◆ Production capacity constraints:  
 $\bar{x} \geq x$
- ◆ Nonnegative price constraints:  
 $p \geq 0$
- ◆ Nonnegative profit constraints:  
 $p \geq Ap$ ; or  
 $[I - A]p \geq 0$
- ◆ Aggregate income:  
 $Y = \text{Profits}$   
 $= ([I - A]p)'x$

# Extensions to Include Land and Labor

- ◆  $l$ , the nonnegative vector of labor coefficients (labor required per unit of the  $i$ th output):

$$L = l'x$$

- ◆ Aggregate labor constraint:

$$\bar{L} \geq L$$

- ◆  $t$ , the nonnegative vector of land coefficients (land required per unit of the  $i$ th output):

$$T = t'x$$

- ◆ Aggregate land constraint:

$$\bar{T} \geq T$$

- ◆ Nonnegative price, wage and land rental constraints:

$$p \geq 0; w \geq 0; r \geq 0;$$

- ◆ Nonnegative profit constraints:

$$p \geq Ap + wl + rt; \text{ or} \\ [I - A]p - wl - rt \geq 0$$

- ◆ Aggregate income:

$$Y = \text{Profits} + \text{Wages} + \text{Land Rents} \\ = ([I - A]p - wl - rt)'x + wL + rT \\ = ([I - A]p)'x$$

# The Determination of Final Demand

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- ◆  $y$ , the vector of final demands, is assumed to depend on aggregate income and to the vector of prices, the wage rate and the land rental rate, but not necessarily linearly. Thus, the final demands of the goods have to be separately modeled as functions of aggregate income and prices.

$$y = F(Y, p, w, r).$$

- ◆ Modeled in this particular way, the final demands include both household and government consumption final demands. It is possible to disaggregate the final demands into its two components, household consumption and government consumption, if necessary.

# The Determination of Final Demand

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- ◆ Without loss of generality, the price of one good, for example, labor, can be chosen as the numeraire good. Subject to this normalization, the prices, which are not normally determined in an input-output model, can be determined in an iterative process. For example, the price of any good whose required output level exceeds the existing capacity can be raised and the price of any good whose required output level is below the existing capacity can be lowered. The wage rate and the land rental rate can also be similarly adjusted.
- ◆ Once a feasible set of final demands and prices can be found, it represents a possible equilibrium for the economy. One can also try to find a feasible solution by maximizing aggregate income with respect to the prices, subject to a price normalization and all the constraints of the standard input-output model.

# Possible Extensions:

## Fixed Input-Output Coefficients

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- ◆ While it is true that input-output coefficients are updated from time to time, endogenous changes in the input-output coefficients within a planning period can and should be taken into account. Input-output coefficients can change because of substitution between inputs. If the relative prices of the goods change, then the optimal combination of inputs to produce a given output will also change. This will result in changes in the input-output coefficients.
- ◆ Input-output coefficients can also change because of technical progress and learning by doing, so that less input is needed per unit output.
- ◆ They can also change because the quality of an input, or of a marginal unit of the input, may have changed. The latter can go either way—the marginal unit can have better quality because it is produced with newer technology, or it can have worse quality, because one has to use lower grade ores.
- ◆ The energy and labor coefficients can also vary widely depending on the type of energy and the educational attainment of the labor.

# Possible Extensions:

## Fixed Input-Output Coefficients

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- ◆ Input-output coefficients can be updated through annual surveys. They can also be modified through the use of trend projection methods, based on the two most recent input-output tables.
- ◆ However, it is impossible to update all the coefficients especially if the number of sectors distinguished is large. (The current Chinese input-output table distinguishes 139 sectors.) Only the critically important coefficients can and need to be updated.
- ◆ An alternative will be to use a separate module to derive both the aggregate quantity supplied and the price of a specific output. An example is the separate modeling of the electricity generation sector, where many different technologies co-exist.

# Possible Extensions:

## Constant Returns to Scale

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- ◆ Typical input-output models assume constant returns to scale, so that if the final demands are doubled, the gross outputs required are also doubled.
- ◆ Economies of scale normally do not exist for material inputs if they are measured in quality-equivalent terms.
- ◆ Economies of scale are mostly found in capital and labor. There are three aspects to the incorporation of the effects of scale. The first is the reduction in costs and the increase in efficiency as the average scale of the plants/firms in the sector is increased. The second is the gain in pricing power and hence profitability of a firm as its share of the market is increased. The third is however the entry of high-cost producer as the aggregate demand increases; for example, the entry of shale oil producers in the U.S. in recent years.

# Possible Extensions:

## Constant Returns to Scale

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- ◆ Research has indicated that the scale effects of the first kind mostly apply to the capital cost, based on the six-tenths factor rule of capital cost (this is an empirically observed rule, and is roughly justified on the basis of the ratio between the total area of the surfaces required to contain a given volume), and occasionally also to labor cost, as in electricity generation plants. If there were economies of scale, expansion of new production capacity can reduce the resource requirement per unit of output capacity.
- ◆ Scale effects of the second kind are difficult to model in the input-output analysis context and depend on marketing efforts, advertisement and goodwill. Scale effects of the third kind can be modeled with the technical coefficients of the sector concerned increasing with the level of gross output of that sector.
- ◆ We propose a solution for the incorporation of scale effects of the first kind below.

# A Dynamic Version of the Leontief Input-Output Model

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- ◆ First, we introduce a dynamic version of the Leontief input-output model.
- ◆ In a dynamic Leontief input-output model, the production capacity of each of the sectors is allowed to expand over time:

$$\bar{x}_{t+1} = \bar{x}_t + k_t$$

- ◆ where  $k$  is a vector of the newly completed production capacity in the  $t$ th period and it is assumed that there is no depreciation or retirement for the sake of simplicity.

# A Dynamic Version of the Leontief Input-Output Model

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- ◆ The investment of  $k_t$  requires inputs from all the sectors in accordance with a matrix of capital coefficients  $B$ , resulting in the investment final demands:

$$i_t = Bk_t$$

- ◆  $i_t$  is the vector of final demands for the outputs of the economy due to investments in additional production capacities.
- ◆ The coefficient  $B_{ij}$  is defined as the quantity of the  $i$ th good required per unit of the increase in the production capacity of the  $j$ th industry.
- ◆  $k_t$  itself may be a function of expected future final demands and hence expected aggregate income, expected prices, including the wage and land rental rates, and the user cost of capital (which depends on the real rate of interest), and must also be separately modeled.

# A Dynamic Version of the Leontief Input-Output Model

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- ◆ The material balance equations become:

$$\begin{aligned}x_t &= Ax_t + y_t + i_t; \\ &= Ax_t + y_t + Bk_t; \\ x_t &= [I - A]^{-1} (y_t + Bk_t)\end{aligned}$$

- ◆ Production capacity constraints:

$$\bar{x}_t \geq x_t$$

- ◆ Aggregate labor and land requirements:

$$L = l'x; T = t'x$$

- ◆ Aggregate labor and land constraints:

$$\bar{L} \geq L; \bar{T} \geq T$$

- ◆ Nonnegative price, wage and land rental constraints:

$$p \geq 0; w \geq 0; r \geq 0;$$

# A Dynamic Version of the Leontief Input-Output Model

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- ◆ Nonnegative variable profit constraints:

$$p \geq Ap + wl + rt; \text{ or}$$

$$[I - A]p - wl - rt \geq 0$$

- ◆ Aggregate income:

$$Y = \text{Profits} + \text{Wages} + \text{Land Rents}$$

$$= ([I - A]p - wl - rt)'x + wL + rT$$

$$= ([I - A]p)'x$$

- ◆ Production capacity expansion:

$$\bar{x}_{t+1} = \bar{x}_t + k_t$$

- ◆ The final demands of  $y_t$  and  $k_t$  will have to be separately modeled.

# Possible Extensions:

## Constant Returns to Scale

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- ◆ Economies of scale in capacity expansion can be modeled by the introduction of a scale coefficient,  $\mu_i$ , for the capacity expansion of each sector.
- ◆  $\bar{x}_{(t+1),i} = \bar{x}_{ti} + k_{ti}^{\mu_i}$
- ◆ For example, if the six-tenths factor rule of capital costs holds, then the scale coefficient will be approximately 1.67 (=1/0.6). These scale coefficients can be estimated sector by sector.
- ◆ For some sectors, for example, electricity generation, there may also be economies of scale in the use of labor, in which case the labor coefficients can be modeled as a non-decreasing function of the aggregate output of that particular sector but with a non-increasing first derivative with respect to the aggregate output.

# Possible Extensions:

## All Goods are Desirable

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- ◆ We know that not all goods produced are desirable. For example, pollutants are undesirable.
- ◆ Carbon dioxide emission, and for that matter the emission of any pollutant, e.g., PM 2.5 particles, can be modeled as a necessary but undesirable by-product of the production process.
- ◆ There are many different ways to incorporate an undesirable good, a pollutant, in an input-output model.
- ◆ One straightforward way is to introduce a vector of pollution coefficients,  $c$ , with each element  $c_i$  as the pollutant produced per unit output of  $x_i$ , similar to the labor or land coefficients.
- ◆ Similarly, a pollutant tax per unit of pollution,  $q$ , can be introduced in the same way as the wage rate and the land rental rate.

# Possible Extensions: An Input-Output Model with an Undesirable Good

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- ◆ Aggregate labor and land requirements:

$$L = l'x; T = t'x$$

- ◆ Aggregate labor and land constraints:

$$\bar{L} \geq L; \bar{T} \geq T$$

- ◆  $c$ , the nonnegative vector of pollutant coefficients (pollutant produced per unit of the  $i$ th output):

$$C = c'x$$

- ◆ Aggregate pollutant constraint:

$$\bar{C} \geq C$$

# Possible Extensions: An Input-Output Model with an Undesirable Good

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- ◆ Nonnegative price, wage, land rental and pollutant tax constraints:

$$p \geq 0; w \geq 0; r \geq 0; q \geq 0;$$

- ◆ Nonnegative profit constraints:

$$p \geq Ap + wl + rt + qc; \text{ or}$$

$$[I - A]p - wl - rt - qc \geq 0$$

- ◆ Aggregate income:

$$Y = \text{Profits} + \text{Wages} + \text{Land Rents}$$

$$= ([I - A]p - wl - rt)'x + wL + rT - qC$$

$$= ([I - A]p)'x - qC$$

- ◆ Aggregate pollution tax revenue, which accrues to the government, is given by:

$$qC$$

# Possible Extensions:

## Introduction of an Amelioration Sector

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- ◆ The model described above only deals with limiting the potential production of pollutants. However, it does not deal with cleaning up the environment, which requires real resources and inputs.
- ◆ The clean-up activities can be modeled with the introduction of an amelioration sector, a production sector that specializes in the reduction of pollutants.
- ◆ However, there is no price for public goods such as clean air and clean water, and therefore there is no nonnegative profit constraint for the amelioration sector. One can impose the condition that the aggregate clean-up costs are financed out of the aggregate pollution tax revenue.

# Possible Extensions:

## Fixed Infrastructure

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- ◆ Often the basic infrastructure of the economy is taken as given and fixed.
- ◆ What happens if there is new infrastructure coming on stream?
- ◆ So all of a sudden there may be alternative sources of supply (and demand), without any change in the production capacities of the sectors. New basic infrastructure may affect the input-output coefficients as well as the final consumption and investment demands.
- ◆ In addition, infrastructure also has to be built and maintained. Thus, perhaps a separate sector for infrastructure may be helpful. However, infrastructure per se cannot in general be expected to be directly profitable or even break even. For example, intra-urban mass transit systems are often subsidized by the government. They generate positive externalities that cannot be captured in the market, i.e., the reduction in pollution and congestion, the saving of time, the conservation of land through enabling higher residential densities for other valuable uses, etc.

# Possible Extensions:

## Homogeneity of Goods

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- ◆ In a strictly technical sense, the non-homogeneity of goods in the same sector can always be overcome by disaggregating the sector further. However, this is not always practical or practicable.
- ◆ A separable module may work—for example, one can append a separate module for the energy sector or even for the electricity generation sector. The module can determine the optimal combination of different forms of energy to be produced and the marginal cost of the supply of energy to the economy. Iteration between the input-output model and the separable module can yield the equilibrium supply and demand of energy and its price on the margin.
- ◆ The distinction between the production of exports from the processing and assembly sector and from the non-processing and assembly sector is another example of how non-homogeneity of goods can be successfully accommodated.

# Proposed Applications for the Chinese Economy

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- ◆ Four possible applications that are topical are proposed here. Many more applications can be considered.
- ◆ (1) The estimation of the quantities (volumes) of Chinese exports and imports
- ◆ (2) The estimation of the value-added of Chinese exports
- ◆ (3) The analysis of the effects of a carbon emission limit on the Chinese economy
- ◆ (4) The analysis of the effects of a carbon tax on the Chinese economy

# The Estimation of the Quantities (Volumes) of Chinese Exports and Imports

- ◆ One important question about the Chinese economy today is whether the rates of growth of the quantities (volumes) of Chinese exports and imports, as opposed to their gross values, have been rising or falling, and by how much. We know that the gross values of Chinese exports and imports have been falling over the past year. These declines in values could be due to the decline in Chinese export prices (driven in part by falling prices of manufactured goods in China), the devaluation of the Renminbi, and the sharp fall in the prices of oil and other natural resources on the World market.
- ◆ Of greater relevance to the Chinese economy is whether in real terms Chinese exports and imports have been rising or falling. It is the real quantity of exports and imports that are more relevant to the determination of Chinese real GDP and employment. For example, with the significant fall in the World price of oil, it is possible for the quantity of oil imports to be rising or stable but for the value of oil imports to be declining significantly.

# The Estimation of the Value-Added of Chinese Exports

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- ◆ Countries with large quantities of exports may actually derive relatively little domestic value-added from their exports. However, it is the quantity of domestic value-added that affects domestic GDP and employment.
- ◆ Moreover, countries also typically export and import large quantities in the same commodity classification—but these are actually different goods and products being exported and imported. For example, China is a large exporter as well as importer of electronics. Almost all Apple iPhone are manufactured in China, but the Chinese domestic value-added content is very low. For another example, Malaysia used to be the World's largest exporter as well as importer of semiconductors because they were the final packagers of finished semiconductors.

# The Estimation of the Value-Added of Chinese Exports

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- ◆ A second important question about the Chinese economy today is therefore whether the value-added, as opposed to gross value, of Chinese exports have been rising or falling, and by how much. The value-added is a much more accurate and relevant measure of the contributions of Chinese exports to Chinese GDP and employment.
- ◆ Through an input-output model that distinguishes exports by sectors, and by whether they are competitive or non-competitive, we can derive the value-added content of the exports in each sector. It is entirely possible for the value of aggregate exports to fall but for the total value-added from aggregate exports to rise in response to greater localization of components and to changes in the sectoral composition of export demands.

# The Estimation of the Value-Added of Chinese Exports

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- ◆ From the value-added of exports from each sector, the aggregate value-added from all exports to the rest of the World and to selected countries or regions such as the U.S., the European Union and the ASEAN region can be easily computed and compared to the trends of exports to these countries/regions.
- ◆ For economic policy purposes, the changes in value-added, especially real value-added, from exports, are much more relevant than the changes in the gross value of exports or even the quantity (volume) of exports.
- ◆ It is also useful to estimate the quantity of employment generated by the exports of today compared to the exports of say ten years ago.

# The Analysis of the Effects of a Carbon Emission Limit on the Chinese Economy

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- ◆ China has agreed to the COP 21 accord in Paris. China has also committed to regulating its total carbon dioxide emission so that it will begin to decline in 2030, with milestone targets in intervening years. Carbon dioxide emission, and for that matter the emission of any pollutant, e.g., PM 2.5 particles, can be analyzed as a necessary by-product of the production process.
- ◆ There are many different ways to incorporate a pollutant in an input-output model.
- ◆ One straightforward way is to introduce a vector of pollution coefficients,  $c$ , with each element  $c_i$ , as the pollutant produced per unit output of  $x_i$ , similar to the labor or land coefficients.  $q$  is the tax per unit of the pollutant.
- ◆ This is an application of the model with an undesirable good.<sup>29</sup>

# The Analysis of the Effects of a Carbon Emission Limit on the Chinese Economy

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- ◆ One may further introduce a pollutant amelioration sector with its output measured in terms of reduction of the quantity of the pollutant produced.
- ◆ There will then be an additional component of the final demand, the planned quantity of reduction of the pollutant.
- ◆ The market price of the “anti-pollutant” can be assumed to be zero (a public good), so that there is no non-negative profit constraint. However, there may be a government subsidy constraint which limits the total loss of the pollutant amelioration sector, perhaps to the total pollution tax collected in the first instance.
- ◆ Finally, it is also possible to include the “reduction” in the quantity of pollutants in the aggregate pollution constraint.
- ◆ It is actually useful to look into the possible anti-pollution technologies and estimate the likely input-output coefficients for the pollutant amelioration sector.

# The Analysis of the Effects of a Carbon Emission Limit on the Chinese Economy

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- ◆ One of the possible instruments for limiting carbon dioxide emission is a carbon tax, as analyzed above. Input-Output analysis can be used to derive the effects, benefits and costs of a carbon tax, and to estimate the magnitude of the tax necessary to achieve the Paris goals.
- ◆ It is also possible to analyze the effects of carbon trading, which is more complicated than a carbon tax because of the problems of compliance and enforcement. But the Chinese Government seems to have chosen carbon trading for the time being. It is of interest to compare the results of the implementation of a carbon tax with those of implementing a carbon trading system.
- ◆ Input-output analysis can be used to analyze alternative scenarios for carbon dioxide emission of China

# Concluding Remarks

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- ◆ There are many other possible applications of input-output analysis in the Chinese economy as the many applications undertaken by Prof. CHEN Xikang and his collaborators in the past have shown.
- ◆ Extension to an open economy is of critical importance in this age of globalization. Here, the distinction between competitive and non-competitive exports and imports is crucial.
- ◆ The construction of international and interregional input-output tables can be timely and useful, especially in the light of the “One Belt, One Road (OBOR)” initiative of China.